A SAFETY CONSIDERATION IN DESIGN OF CONCRETE FLOOR SYSTEMS USING FINITE ELEMENTS[1]

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Concrete floor systems are designed to resist specified loads with a code prescribed safety factor. The two central steps in the design process are: (i) to determine the force "demand" at each location in a floor system caused by the specified loads, and (ii) to provide adequate reinforcement to meet the force demand – namely, to provide the required resistance capacity. The common procedure to determine the force "demand" for complex floors is to use the Finite Element Method (FEM) for the analysis of the structure. Once the structure is analyzed, the design values "demands" are extracted from the FEM solution, using one of the several options. The most commonly used options are based on the "stress integration" or different forms of "nodal integration." First generation FEM programs are based on the stress integration method. Taking advantage of developments in computational technology, the currently available third generation FEM programs use the nodal integration, or improved versions of it (ADAPT-Builder Platform).

The primary reason for leaving behind the stress integration technology, and the emphasis on nodal integration for the extraction of design values has been the concern on the validity of the design values obtained from the former method, in particular when it relates to the safety of complex floor systems.

Using a simple example, this Technical Note illustrates one of the many common cases, where the stress integration method underestimates the design values and leads to reduced safety factor for the structure. Simply, the designs based on the stress integration technology can underestimate the reinforcement necessary to meet the safety requirements of a design, whereas the designs based on the nodal integration method always yield the correct design values, hence meeting the safety requirements of design in all instances . A full account of the two methods is given in ADAPT Technical Note TN302. Appendix A of this Technical Note offers a brief overview.

Another major advantage of designs based on the "nodal integration," is that the solution is not sensitive to the details of subdivision of the structure into finite element cells. A coarse subdivision results in values accurate for design, whereas the design values derived from the stress integration method are highly sensitive to the density and details of the finite element subdivision of the structure used in the analysis [ADAPT TN184].

Consider the slab of uniform thickness shown in Fig. 1. The slab is fixed at one edge and free at the other three edges in form of a cantilever. Selfweight is not considered. To afford simplicity and a condition that can readily be verified by simple hand calculation, without compromising the concept and the procedures, assume a distribution of externally applied load across the opposite edge in the plane of the slab as shown in the figure. Evidently, the load will be transferred through the slab from the point of its application at the free edge of the slab to the supported edge at the other end. For the safe

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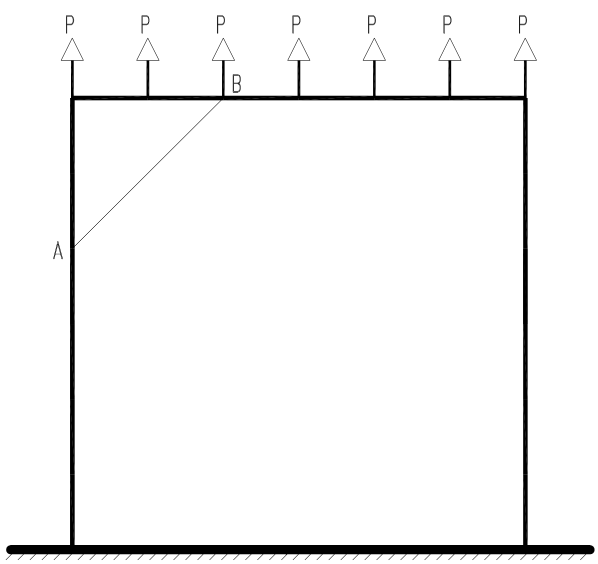
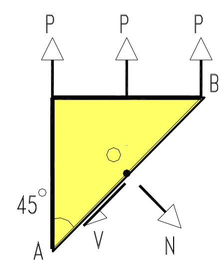
transfer of load, at any section through the slab, the reinforcement provided must be adequate to resist the force that flows through that section. In the following, the determination of the force at a design section (demand) is investigated, using both the “stress integration” and the “nodal integration” methods.

Let the design section selected sever a wedge AB from the corner to the slab as shown in Fig. 1-b. The demand force for the design section AB consists of an axial force N and a shear force V. Using simple statics the axial force N can be determined from the equilibrium of the wedge. Assuming P=10, the axial force N is given by:

$$N = 3*P*\cos(45) = 3 * 10 * 0.707 = 21.21 \quad (1)$$

The minimum amount of reinforcement normal to the section AB for the safety of the structure shall be adequate to resist the force N=21.21.





(b) Wedge of slab between points A and B, identified by “design section” AB and showing the resultants of forces acting on AB

(a) Square slab subjected to in-plane forces P

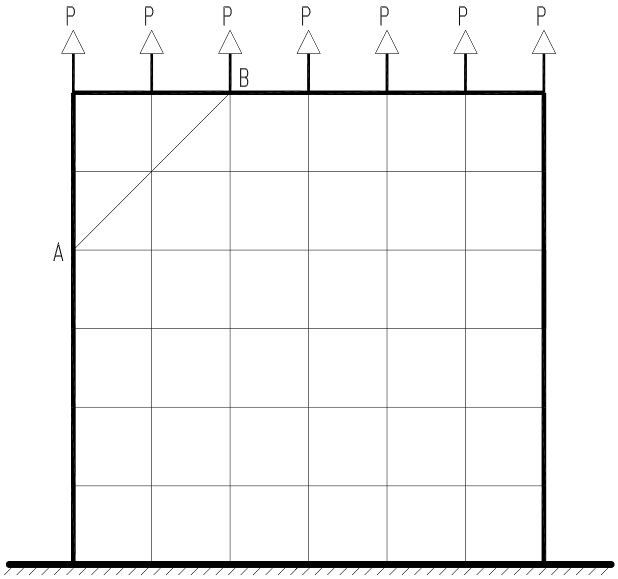
FIGURE 1 SLAB UNDER IN-PLANE LOADS

Using a subdivision of the plate into finite element cells as shown in Fig. 2-a, solutions were obtained using ADAPT-Floor Pro and SAP2000[3] program. The program SAP2000 was used to determine the axial force N on design section AB, using the “stress integration” method. ADAPT-Floor Pro determines the actions at the centroid of a design section using the “nodal integration” method. The results obtained from the two procedures are also entered in Fig. 2b.

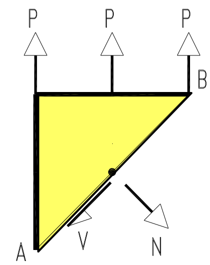
Correct value	21.21	Using statics
ADAPT Floor-Pro	21.21	Using nodal integration
SAP 2000 stress integration	16.86	Using stress integration

Note that in this instance, the “stress integration” method underestimates the required force by about 20%. Also, note that SAP2000 would have also given the correct value, if the “nodal integration” option were used.

[3] SAP 2000 is a product of CSI



(a) Subdivision of slab into finite element cells



(b) Axial force N calculated using different methods

N = 21.21 Correct value using “statics”
N = 21.20 Using “nodal integration” (Floor Pro)
N = 16.86 Using “stress integration”

FIGURE 2 SUBDIVISION OF SLAB INTO FINITE ELEMENT CELLS AND THE VALUES FOR DESIGN SECTION AB

This example clearly illustrates that where a design section is close to a concentrated load, the stress integration procedure grossly underestimates the design values it reports. This situation is not uncommon in the vicinity of columns and concentrated line and point loads that are significant in transfer plates carrying a multitude of levels. If the sections are far away from the concentrated loads, the stress integration method is likely to overestimate the design values. One reason in underestimating the design values is that in its integration a portions of the sphere of stress influence from concentrated loads and discontinuities that falls beyond the design section are not included in the integration. Figure 3 illustrated the diffusion of axial stress into the slab for the loads shown in Fig. 1

FIGURE 3 DISTRIBUTION OF AXIAL STRESS IN UP-DOWN DIRECTION FROM THE LOADS SHOWN IN FIG.1

The use of the stress integration method is where slabs are of uniform thickness and regular geometry, loads are uniformly distributed and the design sections that are close to midspans. The design methods based on stress integration are likely to underestimate the negative moments at the face of supports, leading to a smaller factor of safety at these locations.

As it is outlined in detail in Technical Note TN302[4] the design values reported by the nodal integration method are ALWAYS SAFE, since they are always in equilibrium with the applied loads. The nodal integration method is particularly suited for floor systems with complex geometry, such as irregular boundary and support layout, steps openings, and concentrated or line loads. Unlike for the stress integration procedure, regardless of proximity of a design section to a step or other irregularities, the design values reported for a design section using nodal integration procedure are always correct.

Other features of the nodal integration procedure are:

- Mesh density
  The nodal integration procedure is not sensitive to the fineness of the finite element mesh used in analyzing the structure. The design values are “practically” independent of mesh density. Refer to ADAPT Technical Note TN184 (Mesh Density and Accuracy of Design Values) for a detailed account of this topic. In contrast, values obtained from the stress integration procedure highly dependent on the mesh size, shape and density of the finite element analysis used. Generally different design values are obtained, using different mesh densities and layout. This necessitates the “validation” of design values based on stress integration – a step that is not necessary when using nodal integration.

- Twisting Moment Mxy and Wood-Armer
  When using the stress integration method, the question arises whether or not the twisting moment Mxy at the level of infinitesimal elements should be considered to be contributory to the integrated design values reported. Wood-Armer approach is one approximation for handling this question. Using the nodal integration method, the question does not arise, since the method correctly accounts for all the forces that are contributory to the demand actions at a design section. The method correctly reports the deign values, regardless of whether the torsional parameters are included in the finite element formulation or not.

CONCLUDING REMARMS

For design of complex floor systems that feature irregular geometry, multiple steps, openings, and small span to depth ratios, such as transfer plates, it is prudent to use design tools that are based on nodal integration technology or its extensions, in order to arrive at guaranteed safe solutions.

The design values that are obtained from software based on stress integration are almost never in equilibrium with the applied loads. The values reported for design sections in the vicinity of concentrated loads and columns are practically always less than required for the equilibrium of forces. Hence, they compromise the design-intended safety of the structure.

Unlike for the nodal integration option, the values obtained from the stress integration method of first generation FEM software highly depend on the density and shape of the finite element cells used in the analysis.

[4] ADAPT TN302 Evaluation of Design Values at Design Sections Using Floor Pro

APPENDIX A

OPTIONS FOR DETERMINATON OF DESIGN VALUES AT DESIGN SECTION

The following is a brief excerpt from the ADAPT Technical Note TN302. For the full account of the topic refer to the original Technical Note.

The reinforcement at a design section, such as AB shown in Fig. A-1 is based on the forces (moments, shears, etc) that act on the section and have to be resisted for the safety of the structure.

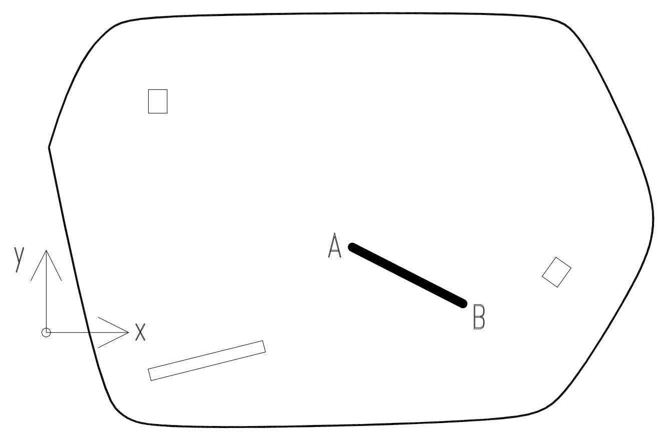


FIGURE A-1 PLAN OF A SLAB, SHOWING DESIGN SECTION AB

One option, referred to as “stress integration,” for calculating the force demand on a design section, such as AB in Fig. A-2, is to assume that the force is the resultant of the stresses on the infinitesimal elements along the design section. Thus, integrating the local stresses along the length of a design section a force demand is obtained based on “stress integration.” Obviously, among other factors, the value of the resultant force depends on the accuracy of the local stresses. It is a well known principle in the finite element formulation that in the general case the internal stresses in the elements are not accurate, hence their summation cannot be accurate either.

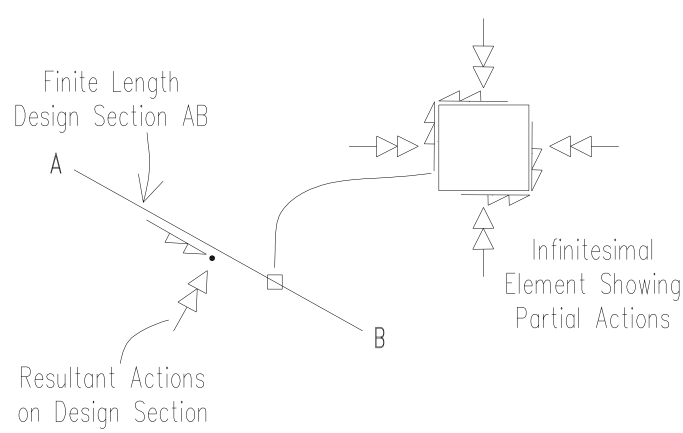


FIGURE A-2 STRESS INTEGRATION OPTION

Another option is to base the calculation of the force resultant on a design section, such as AB in Fig. A-3 on the static equilibrium of the forces that the design section has to resist. Isolating a region of slab bounded by the finite element cells adjoining the design section, the nodal forces (F) along the perimeter of the isolated region and the externally applied load (P) on the isolated region are the only actions that are contributory to the demand at the design section and have to be resisted by the design section. Using statics the value of actions at the design section can be calculated. It is important to note that since in FEM the nodal forces are in equilibrium with the applied loads, the solutions based on this method are always safe.

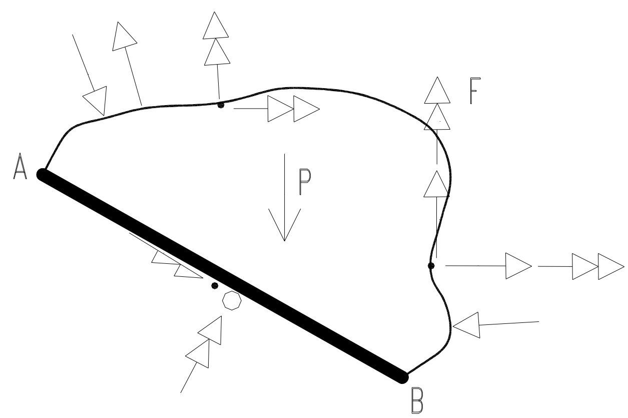


FIGURE A-3 EXTENDED NODAL INTEGRATION OPTION FOR DESIGN SECTION AB.